Reverse Engineering a Nomograph to Equations

by
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Introduction

Engineers often rely on engineering data that is presented in nomographs to perform engineering calculations. These graphs are useful for presenting a correlation between a physical property such as viscosity or diffusivity as a function of a measured parameter such as temperature (see Figure 1).[1] This was especially appropriate when these calculations were complex and computers for performing these calculations were not readily available. However, with the widespread use of personal computers and powerful computational software, using a graphical method to calculate these data is not only cumbersome, but also inefficient. It would be much more useful to have equations to represent the data mathematically. This article presents a method for "reverse engineering" the data presented in the nomograph to equations that can be used for computations.

Figure 1 is an example of a special type of nomograph known as a Line Coordinate chart.[2] In this nomograph, the values of the independent and dependent variables are related by a pivot point. A straight line drawn through the pivot point will give the viscosity as a function of temperature for a particular gas. Each gas has a different pivot point with different coordinates as shown in Table 1.

The form of the equation for a Line Coordinate nomograph is given by

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^n \qquad (1)$$

where μ = gas viscosity
μ_0 = reference gas viscosity
T = temperature
T_0 = reference temperature
n = exponent

All Line Coordinate nomographs have this same functional form.

The key to reverse engineering this nomograph lies in the fact that the reference viscosity and temperature (μ_0 and T_0) are functions of the y coordinate only and the exponent (n) is a function of the x coordinate only. Thus, it is relatively easy to determine a functional relationship between the gas viscosity and temperature that will fit the form of Equation (1) using the

coordinates of the pivot point as input data. To reverse engineer the nomograph, we need to be able to obtain empirical functional relationships for the reference viscosity and temperature (μ_0 and T_0) and the exponent (n) as functions of the pivot point coordinates (x and y). We can do this by taking values from the nomograph and using data regression analysis software to determine these functional relationships.

Reverse Engineering Method

As an example of how to reverse engineer a nomograph, consider Figure 1. The functional dependence of the reference viscosity and reference temperature (μ_0 and T_0) on y is determined by drawing a number of horizontal lines on the chart at different values of y. This gives the values shown in Table 2 for the reference temperature and viscosity as a function of y. Since these values are obtained by drawing horizontal lines, they are not a function of x.

Any number of empirical correlations could be developed for μ_0 and T_0 as a function of y. The form of the equations could be linear, polynomial, logarithmic, power law, or exponential. The particular form of the equations is not as important as their accuracy in representing μ_0 and T_0 as functions of y. Most spreadsheet software (such as Excel™ or Quattro Pro™) includes data regression analysis routines that can be used to determine the parameters for the correlating equations. The accuracy of the fitted equations is indicated by the standard statistical parameter, the Coefficient of Determination (R^2).* Generally speaking, the closer R^2 is to one, the better the data are represented by the correlation. The regression analysis routines in Excel™ and Quattro Pro™ calculate and report R^2 as part of their normal output.

Using the values in Table 2, regression analysis gives the following empirical correlations for μ_0 and T_0 as functions of y:

$$\mu_0[\text{cP}] = \exp\left[-5.0471 + 0.06550\,y\right] \qquad (R^2 = 0.9998) \qquad (2)$$

$$T_0[\text{K}] = 1059.1 - 41.35\,y + 0.5325\,y^2 \qquad (R^2 = 0.9997) \qquad (3)$$

It should be noted that the correlation of the reference temperature (T_0) uses absolute temperature units. All temperature values used for deriving these correlations were converted to absolute temperature before the calculations proceeded. This was done for two reasons: first, the kinetic theory of gases shows that low-pressure gas viscosity is a function of absolute temperature,[3] and second, to avoid numerical difficulties when deriving the correlations (i.e. taking the logarithm of zero or a negative number). Figure 2 compares these equations with the values from Table 2. As might be expected from the calculated values of R^2, the fit is very good.

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* The Coefficient of Determination is defined as the proportion of total variability among the dependent variable values that is accounted for by the independent variable values. It is calculated from the formula

$$R^2 = \frac{SS_{yy} - SSE}{SS_{yy}} \qquad 0 \le R^2 \le 1$$

For more information, consult any comprehensive statistics textbook.

Now we need to determine the functional dependence of the exponent, *n*, on *x*. Again, we draw straight lines on the nomograph through various pivot points, *x* and *y*. This gives the viscosity as a function of temperature for each of these pivot points. The values used to reverse engineer Figure 1 are shown in Table 3. Since the form of the equation (1) is exponential, the linearized form is given by

$$\ln(\mu) = n\ln(T) + C \qquad (4)$$

After converting the viscosity and temperature values to logarithmic form, each set of values for a given pair of coordinates *x* and *y* was analyzed using the linear regression routines that come with the Excel™ spreadsheet software. The resulting values of *n* and R^2 for each pivot point is shown in Table 3. Allowing for errors in reading the nomograph, we can see from Table 3 that the exponent (*n*) is indeed a function of *x* only.

Now we need to obtain a relationship between *n* and *x*. Figure 3 shows the calculated values of *n* as a function of *x* for the five different values of the *y* coordinate. Again, the data regression analysis routine in Excel™ was used to fit the seventeen values of *n* from Table 3 as a function of *x*. Figure 3 also shows the two best empirical correlations for *n*; a second-order polynomial, and an exponential equation. These equations are:

$$n = 2.9284 - 0.2992x + 0.008429x^2 \qquad (R^2 = 0.99936) \qquad (5)$$

$$n = \exp[3.5072 - 1.1180x] \qquad (R^2 = 0.99931) \qquad (6)$$

From a statistical standpoint, both of these equations provide an equally good fit for *n* as a function of *x*, especially over the range of expected values of *x*. I have arbitrarily chosen the polynomial form (Equation (5)) to reverse engineer the gas viscosity nomograph. Substituting Equations (2), (3), and (5) into Equation (1) yields an equation that gives gas viscosity as a function of temperature and the various gas coordinates in Table 1:

$$\mu\,[\text{cP}] = \exp[-5.0471 + 0.0655y]\left(\frac{T\,[\text{K}]}{1059.1 - 41.35y + 0.5325y^2}\right)^{\{2.9284 - 0.2992x + 0.008429x^2\}} \qquad (7)$$

The accuracy of this equation depends on two factors – 1) the accuracy of the reverse engineered equation in representing the nomograph, and 2) the accuracy of the original nomograph in representing actual gas viscosity data. Figure 4 shows a comparison of the values calculated by Equation (7) to the values obtained from the nomograph in Table 3. The agreement is excellent; the average deviation is 1.36% while the minimum deviation is 0.004% and the maximum is 5.71%. This is well within the accuracy that can be expected from reading a nomograph. Hence, we have achieved a satisfactory reverse engineering of this nomograph to a relatively simple equation.

The accuracy of the nomograph in representing actual gas viscosity data[4,5,6] is shown in Figure 5. Here, the measured viscosity for several different gases is compared to the values calculated with

Equation (7). The agreement is very good; the average deviation is 1.34%; the minimum deviation is 0.03% and the maximum is 14.23%.

Conclusion

Reverse engineering of a nomograph is a relatively simple process that can pay big dividends in time saved and ease in performing engineering calculations. In no more than a few hours, you can derive an equation that will let you calculate the data you need with minimal input and with good accuracy. Once programmed, you won't need to make hard copies of your nomographs to draw on and you will be able to perform your calculations knowing that any change in your input parameters will automatically calculate the values you need. In today's world, use of hard copy properties correlations is archaic. Reverse engineering a nomograph to equations allows you to preserve the data that has been gathered over the years and use it with 21st century tools.

Nomenclature

n = exponent in Equation (1) [---]
R^2 = statistical Coefficient of Determination, $0 \le R^2 \le 1$ [---]
T = temperature [K]
T_0 = reference temperature [K]
x = coordinate on Line Coordinate nomograph [---]
y = coordinate on Line Coordinate nomograph [---]
μ = gas viscosity [cP]
μ_0 = reference gas viscosity [cP]

References

[1]R. H. Perry and D. W. Green, *Perry's Chemical Engineer's Handbook on CD-ROM*, McGraw Hill Companies, 1999, p 2-321.

[2]D. S. Davis, *Empirical Equations and Nomography*, McGraw Hill Book Company, New York, NY, 1943, p 140.

[3]R. B. Bird, W. E. Stewart and E. N. Lightfoot, *Transport Phenomena, 2nd Ed.*, John Wiley and Sons, Inc., New York, NY, 2002, p 23.

[4]R. C. Reid, J. M. Prausnitz, B. E. Poling, *The Properties of Gases and Liquids, 4th Ed.*, McGraw Hill Book Co., New York, NY, 1987, pp 398-399.

[5]R. H. Perry and D. W. Green, *Perry's Chemical Engineer's Handbook on CD-ROM*, McGraw Hill Companies, 1999, p 2-320.

[6]No author, Viscosity Data for Steam, http://pump.net/otherdata/viscsteamwater.htm

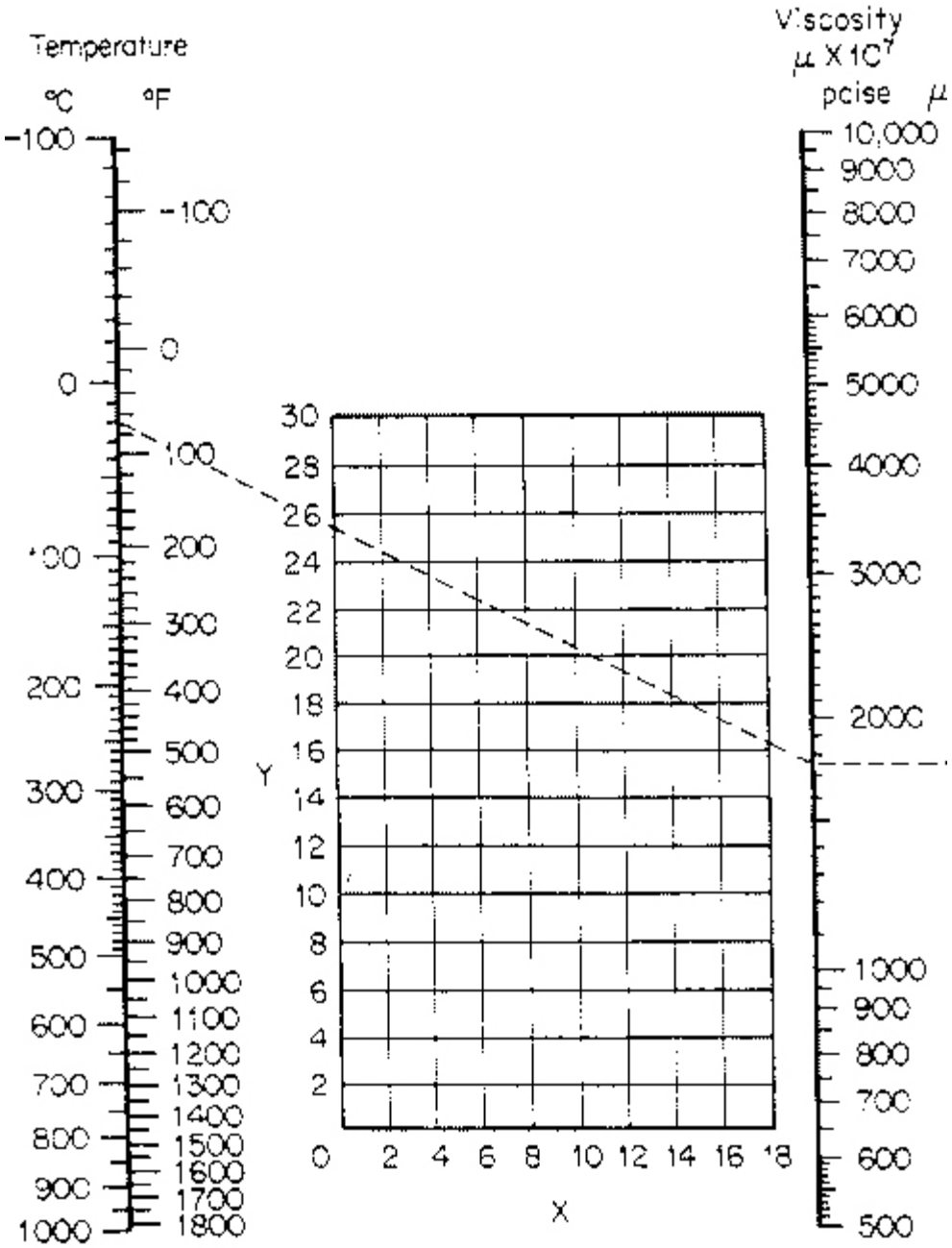

Figure 1. Nomograph for determining the absolute viscosity of a gas as a function of temperature at near ambient pressure (from Perry's Handbook[1], page 2-321). For coordinates, see Table 1.

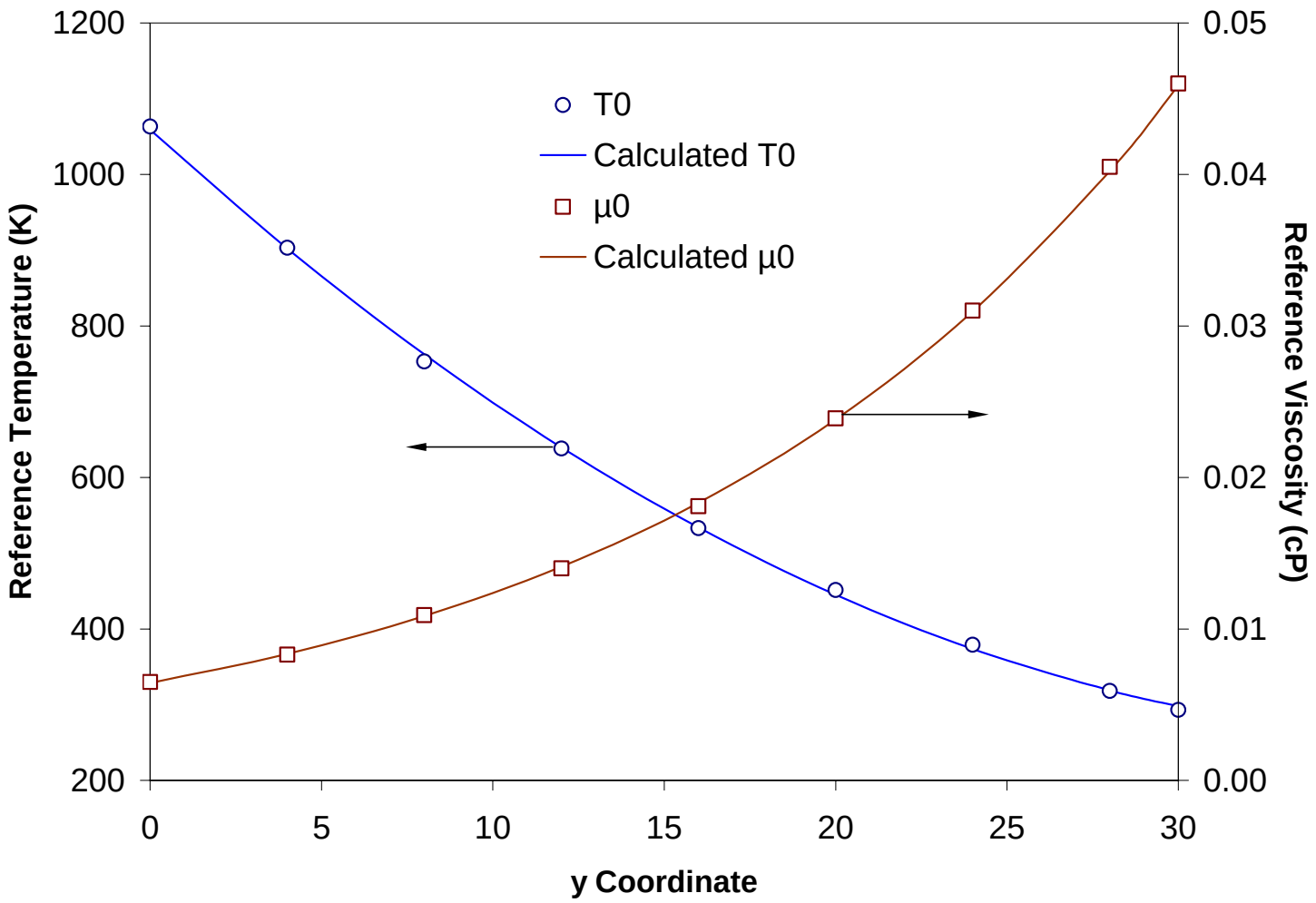

Figure 2. Correlation of the reference temperature and viscosity (T_0 and μ_0) in Equation (1) with the y coordinate on the Gas Viscosity Nomograph (Figure 1). Values are from Table 2.

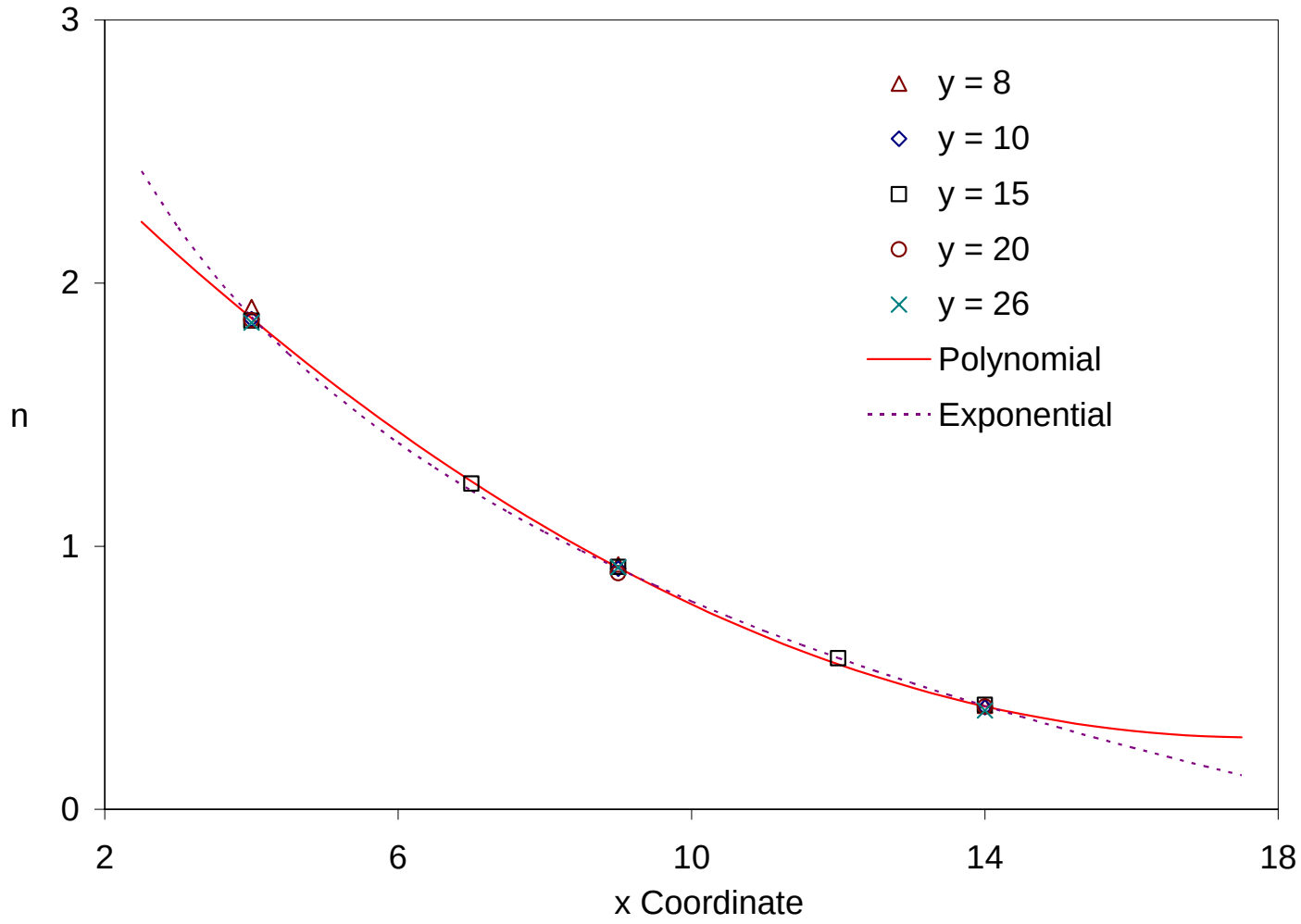

Figure 3. Correlation of the exponent (*n*) in Equation (1) with the *x* coordinate on the Gas Viscosity Nomograph (Figure 1). Values are from Table 3.

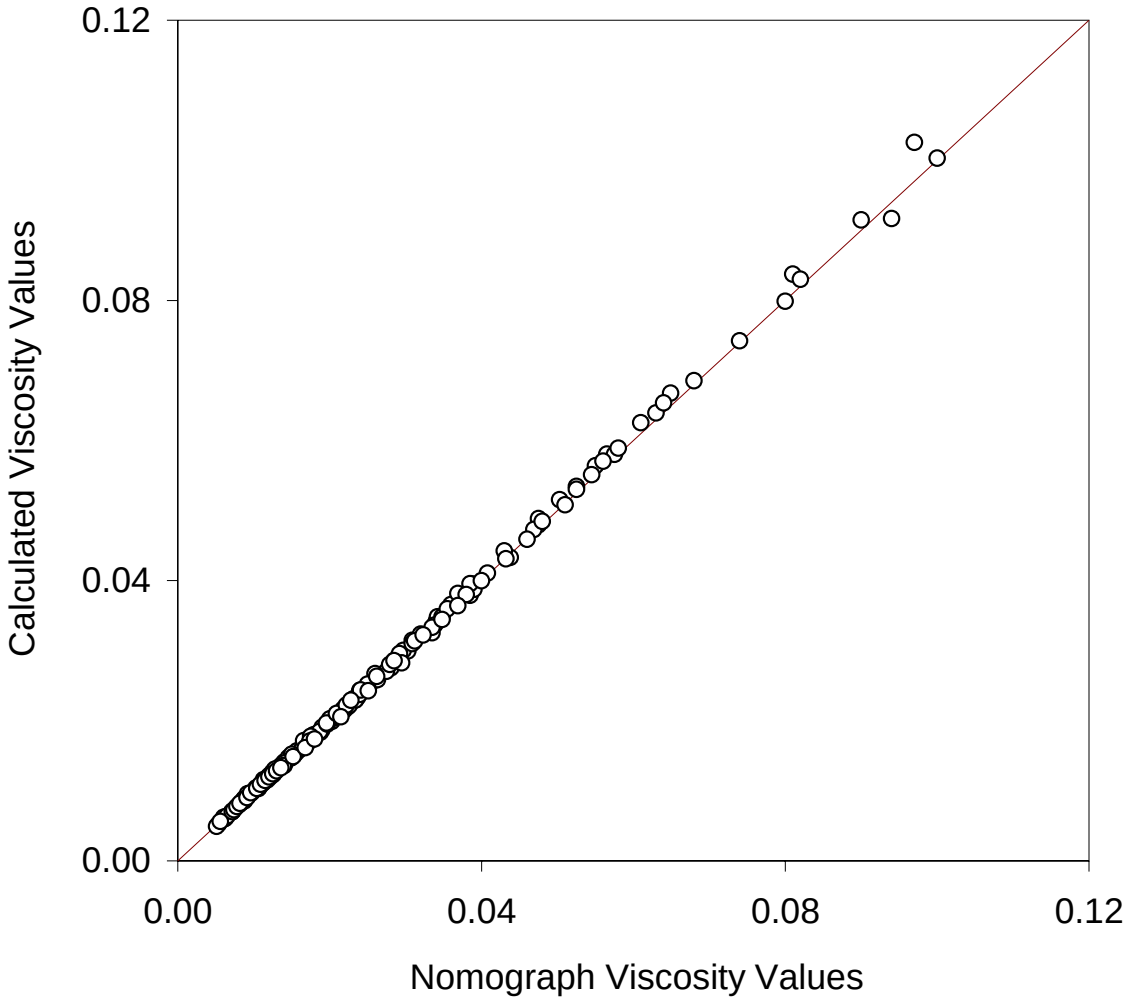

Figure 4. Comparison of calculated viscosity values to the nomograph viscosity values from Table 3.

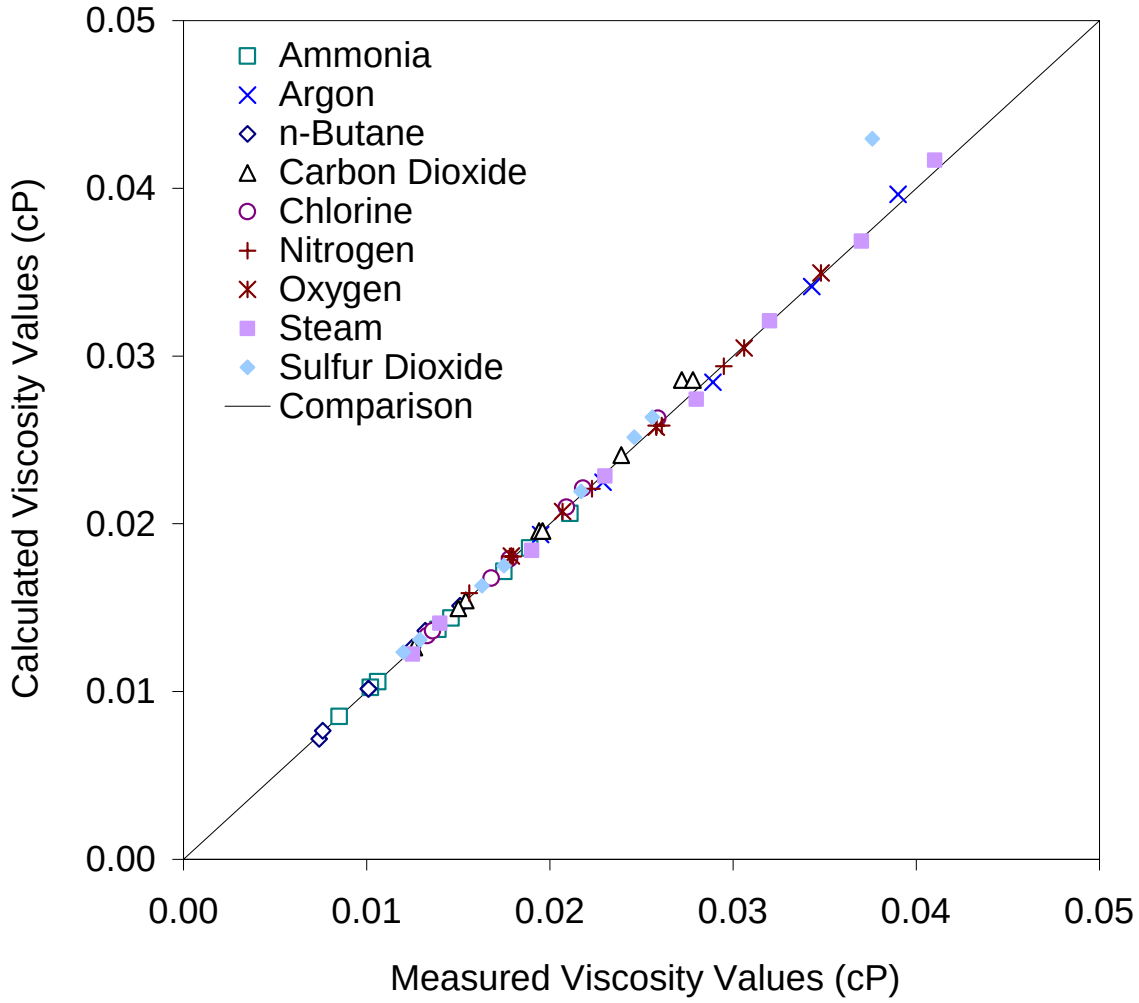

Figure 5. Comparison of calculated viscosity values to measured viscosities at low pressure for various gases. The measured values cover a temperature range from 280 – 1089 K (7 – 816°C).

Table 1. Viscosities of Gases at Low Pressure: Coordinates for Use with Figure 1 (Adapted from Perry's Handbook on CD-ROM., p 2-320)[5]

Gas	*x*	*y*
Acetone	8.4	13.2
Acetylene	9.3	15.5
Air	10.4	20.4
Ammonia	8.4	16.0
Argon	9.7	22.6
Benzene	8.7	13.2
Bromine	8.8	19.4
Butane (*n*)	8.6	13.2
Carbon dioxide	8.9	19.1
Carbon tetrachloride	8.0	15.3
Chlorine	8.8	18.3
Deuterium	11.0	16.2
Ethane	9.0	14.5
Ethylene	9.5	15.2
Helium	11.3	20.8
Heptane (*n*)	8.6	10.6
Hexane (*n*)	8.4	12.0
Hydrogen	11.3	12.4
Methane	9.5	15.8
Nitrogen	10.6	20.0
Oxygen	10.2	21.6
Pentane (*n*)	8.5	12.3
Pentane (iso)	8.9	12.1
Propane	8.9	13.5
Propylene	8.5	14.4
R-11	8.6	16.2
R-12	9.0	17.4
R-21	9.0	16.7
R-22	9.0	17.7
Sulfur dioxide	8.4	18.2
Water vapor	7.6	16.0

Table 2. Reference temperature and gas viscosity data from Figure 1 for various values of the coordinate *y*.

y	T_0 (°C)	T_0 (K)	μ_0 (cP)
0	790	1063	0.0065
4	630	903	0.0083
8	480	753	0.0109
12	365	638	0.0140
16	260	533	0.0181
20	178	451	0.0239
24	106	379	0.0310
28	45	318	0.0405
30	20	293	0.0460

Table 3. Values of gas viscosity from Figure 1 as a function of temperature for various pivot point coordinates, *x* and *y*. Also shown are the corresponding values of *n* and R^2 for various values of the pivot point coordinates.

	y =	8			10			15					20			26		
	x =	4	9	14	4	9	14	4	7	9	12	14	4	9	14	4	9	14
T (°C)	T (K)	μ (cP)	μ (cP)	μ (cP)	μ (cP)	μ (cP)	μ (cP)	μ (cP)	μ (cP)	μ (cP)	μ (cP)	μ (cP)	μ (cP)	μ (cP)	μ (cP)	μ (cP)	μ (cP)	μ (cP)
0	273			0.0074		0.0054	0.0086		0.0072	0.0088	0.0113	0.0129	0.0092	0.0150	0.0196	0.0228	0.0285	0.0323
50	323		0.0051	0.0078		0.0063	0.0092	0.0061	0.0087	0.0103	0.0125	0.0140	0.0128	0.0175	0.0209	0.0312	0.0335	0.0348
100	373		0.0056	0.0082		0.0071	0.0097	0.0082	0.0105	0.0118	0.0138	0.0149	0.0166	0.0200	0.0222	0.0408	0.0380	0.0369
200	473		0.0072	0.0091	0.0061	0.0087	0.0108	0.0127	0.0141	0.0146	0.0157	0.0164	0.0260	0.0250	0.0241	0.0630	0.0469	0.0400
300	573	0.0065	0.0084	0.0096	0.0088	0.0106	0.0115	0.0179	0.0177	0.0175	0.0175	0.0176	0.0369	0.0297	0.0262	0.0900	0.0550	0.0432
400	673	0.0086	0.0097	0.0104	0.0118	0.0122	0.0121	0.0240	0.0218	0.0204	0.0190	0.0187	0.0503	0.0342	0.0279		0.0640	0.0460
500	773	0.0113	0.0111	0.0109	0.0151	0.0140	0.0129	0.0309	0.0259	0.0232	0.0209	0.0197	0.0649	0.0385	0.0292		0.0740	0.0480
600	873	0.0141	0.0126	0.0115	0.0190	0.0154	0.0135	0.0388	0.0303	0.0260	0.0224	0.0206	0.0810	0.0430	0.0309		0.0820	0.0510
700	973	0.0174	0.0139	0.0120	0.0234	0.0170	0.0140	0.0476	0.0341	0.0285	0.0237	0.0217	0.0970	0.0475	0.0320		0.0940	0.0525
800	1073	0.0215	0.0152	0.0125	0.0281	0.0188	0.0147	0.0565	0.0390	0.0311	0.0248	0.0226		0.0525	0.0338		0.1000	0.0545
900	1173	0.0251	0.0168	0.0130	0.0335	0.0203	0.0151	0.0680	0.0438	0.0340	0.0263	0.0234		0.0575	0.0348			0.0560
1000	1273	0.0295	0.0180	0.0136	0.0385	0.0219	0.0158	0.0800	0.0475	0.0360	0.0275	0.0239		0.0610	0.0355			0.0580
	n =	1.9091	0.9314	0.3941	1.8628	0.9146	0.3878	1.8573	1.2378	0.9216	0.5740	0.3972	1.8588	0.8980	0.3911	1.8495	0.9191	0.3756
	R^2 =	0.9995	0.9990	0.9980	0.9999	0.9998	0.9990	0.9998	0.9999	0.9998	0.9994	0.9990	0.9999	0.9980	0.9992	1.0000	0.9990	0.9992

Table 6. Measured and Calculated values of viscosity data for various gases. (Also shown graphically in Figure 5).

Measured vs Calculated values for gas viscosity data:

n-Butane		Carbon Dioxide		Chlorine		Sulfur Dioxide		Nitrogen		Ammonia		Argon		Oxygen		Steam	
x =	8.6	x =	8.9	x =	8.8	x =	8.4	x =	10.6	x =	8.4	x =	9.7	x =	10.2	x =	7.6
y =	13.2	y =	19.1	y =	18.3	y =	18.2	y =	20	y =	16	y =	22.6	y =	21.6	y =	16.0
Measured Values																	
T (K)	μ (cP)	T (K)	μ (cP)	T (K)	μ (cP)	T (K)	μ (cP)	T (K)	μ (cP)	T (K)	μ (cP)	T (K)	μ (cP)	T (K)	μ (cP)	T (K)	μ (cP)
280	0.0074	310	0.0154	293	0.0133	283	0.0120	300	0.0178	310	0.0106					373	0.0125
400	0.0101	400	0.0194	373	0.0168	373	0.0163	500	0.0258	420	0.0146					421.89	0.0140
540	0.0132	600	0.0272	473	0.0209	573	0.0246	250	0.0156	540	0.0189					533	0.0190
300	0.0076	250	0.0126	300	0.0136	973	0.0376	300	0.0180	250	0.0085	250	0.0195	250	0.0179	644.11	0.0230
400	0.0101	300	0.0150	400	0.0178	300	0.0129	400	0.0223	300	0.0102	300	0.0229	300	0.0207	755.2	0.0280
500	0.0125	400	0.0196	500	0.0218	400	0.0175	500	0.0261	400	0.0139	400	0.0289	400	0.0258	866.33	0.0320
600	0.0151	500	0.0239	600	0.0259	500	0.0217	600	0.0295	500	0.0175	500	0.0343	500	0.0306	977.44	0.0370
		600	0.0278			600	0.0256			600	0.0211	600	0.0390	600	0.0348	1088.6	0.0410
Calculated Values																	
μ0 =	0.0153	μ0 =	0.0225	μ0 =	0.0213	μ0 =	0.0212	μ0 =	0.0238	μ0 =	0.0183	μ0 =	0.0282	μ0 =	0.0265	μ0 =	0.0184
T0 =	606.07	T0 =	463.59	T0 =	480.74	T0 =	482.93	T0 =	445.11	T0 =	533.83	T0 =	396.58	T0 =	414.4	T0 =	532.65
n =	0.9786	n =	0.9331	n =	0.9481	n =	1.0098	n =	0.7039	n =	1.0098	n =	0.8191	n =	0.7534	n =	1.1447
T (K)	μ (cP)	T (K)	μ (cP)	T (K)	μ (cP)	T (K)	μ (cP)	T (K)	μ (cP)	T (K)	μ (cP)	T (K)	μ (cP)	T (K)	μ (cP)	T (K)	μ (cP)
280	0.0072	310	0.0154	293	0.0133	283	0.0123	300	0.0180	310	0.0106					373	0.0122
400	0.0102	400	0.0196	373	0.0168	373	0.0163	500	0.0259	420	0.0144					421.89	0.0141
540	0.0136	600	0.0286	473	0.0210	573	0.0252	250	0.0159	540	0.0185					533	0.0184
300	0.0077	250	0.0126	300	0.0136	973	0.0430	300	0.0180	250	0.0085	250	0.0194	250	0.0181	644.11	0.0229
400	0.0102	300	0.0150	400	0.0179	300	0.0131	400	0.0221	300	0.0102	300	0.0225	300	0.0207	755.2	0.0274
500	0.0126	400	0.0196	500	0.0221	400	0.0175	500	0.0259	400	0.0137	400	0.0284	400	0.0258	866.33	0.0321
600	0.0151	500	0.0241	600	0.0263	500	0.0219	600	0.0294	500	0.0172	500	0.0341	500	0.0305	977.44	0.0368
		600	0.0286			600	0.0264			600	0.0206	600	0.0396	600	0.0350	1088.6	0.0417
Error																	
	3.14%		0.18%		0.21%		2.86%		1.38%		0.11%						2.15%
	0.61%		0.88%		0.27%		0.07%		0.21%		1.45%						0.59%
	3.26%		5.03%		0.41%		2.29%		1.75%		1.88%						3.14%
	0.89%		0.17%		0.21%		14.23%		0.25%		0.25%		0.75%		0.99%		0.62%
	0.61%		0.25%		0.58%		1.48%		0.92%		0.43%		1.87%		0.19%		2.05%

1.13%	0.15%	1.47%	0.03%	0.94%	1.46%	1.58%	0.16%	0.29%
0.07%	0.84%	1.52%	1.05%	0.36%	1.95%	0.44%	0.41%	0.42%
	2.77%		2.97%		2.24%	1.66%	0.46%	1.66%