

(More) Random Number Generators for Calculators

By

Namir C. Shammas

Three Sets of Algorithms

- The trigonometric algorithms.
- The single-seed nested exponential algorithms.
- The multiple-seeds nested exponential algorithms.

Using a Penalty Factors

- I use a Penalty factors with 10,000 random numbers iterated 5000 times in 1000 runs. Each run uses different constants.
- The Penalty Factor is a weighed sum of various statistical test scores—closeness of mean to 0.5, closeness of sdev to 0.288, autocorrelation, histogram bin chi-square, sign change, etc.
- I have used the same algorithm to calculate the Factors in previous HHC presentations.

Using Trigonometric Functions

- Uses constants A (and sometimes B) and S—slightly above 1. A and B are randomly selected for each run.
- Uses progressive $\sum$ random numbers generated—**new *heretic* concept!**
- Uses $\sin(x)$ and $\cos(x)$ functions.
- Uses one to three initial seeds.

$$\text{seed2} = \text{frac}(A / (0.5 * (\sin(\text{sm} * \text{seed2}) + \cos(\text{sm} * \text{seed1})) + S))$$

- Ranks #1 among trig algorithms. Uses two initial seeds
- $A = 2,120,625$
- $S = 2 * 0.50000001234$
- $\text{sm} = \text{progressive } \sum \text{ random numbers generated}$
- Obtains Factor = 147.184178, sdev = 11.653767

Code for seed2 =
$$\text{frac}(A/(0.5*(\sin(sm*seed2)+\cos(sm*seed1))+S))$$

- Initialization
 - seed1 = initial seed
 - seed2 = initial seed
 - sm = seed1 + seed2
 - A = 2120625
 - S = 2*0.5000001234

Code for seed2 =
$$\text{frac}(A/(0.5*(\sin(sm*seed2)+\cos(sm*seed1))+S))$$

- seed2 represents the current seed
- seed20 is set equal to seed2
- seed2 is recalculated as $\text{frac}(A / (0.5 \text{ times } (\sin(sm * seed2) \text{ plus } \cos(sm * seed1)) + S))$
- sm is updated by adding seed2
- seed1 is then updated to the value of seed20

$$\text{seed2} = \frac{A}{0.5 * (\sin(\text{sm} * \text{seed2}) + \cos(\text{sm} * \text{seed1})) + S})$$

- Ranks #2 among trig algorithms. Uses two initial seeds
- $A = 2,162,386$
- $S = 2 * 0.50000001234$
- $\text{sm} = \text{progressive } \sum \text{ random numbers generated}$
- Obtains Factor = 147.212109, sdev = 11.499836

$$\text{seed2} = \text{frac}\left(\frac{A}{0.5 * (\sin(\text{sm} * \text{seed2}) + (\sin(\text{sm} * \text{seed1}))^2) + S}\right)$$

- Ranks #3 among trig algorithms. Uses two initial seeds
- $A = 2,120,593$
- $S = 2 * 0.5000001234$
- $\text{sm} = \text{progressive } \sum \text{ random numbers generated}$
- $\text{Factor} = 147.2501, \text{ sdev} = 11.726054$

$$\text{seed2} = \text{frac}\left(\frac{A}{0.5 * (\sin(\text{sm} * \text{seed2}) + \cos(\text{sm} * \text{seed1})) + S}\right)$$

- Ranks #4 among trig algorithms. Uses two initial seeds
- $A = 2,085,438$
- $S = 2 * 0.50000001234$
- $\text{sm} = \text{progressive } \sum \text{ random numbers generated}$
- $\text{Factor} = 147.2623$, $\text{sdev} = 11.600155$

$$\text{seed} = \text{frac} \left(\frac{A}{0.5 * \sin(\text{sm} * \text{seed2})} / (0.5 * \cos(\text{sm} * \text{seed1}) + S) \right)$$

- Ranks #5 among trig algorithms. Uses one initial seed
- $A = 2,056,210$
- $S = 0.5000001234$
- $\text{sm} = \text{progressive } \sum \text{ random numbers generated}$
- $\text{Factor} = 147.283952, \text{ sdev} = 11.661668$

<i>A</i>	<i>Mean</i>	<i>Sdev</i>	<i>Min</i>	<i>Max</i>	<i>Range</i>	<i>CI</i>	<i>High</i>	<i>Low</i>	<i>Max Iters</i>	<i>Max Pop</i>
2120153	147.3416	11.69018	113.7398	203.2851	89.54523	0.324108	147.6657	147.0175	5000	10000
2078284	147.3464	11.62291	113.5701	196.8308	83.26074	0.322243	147.6686	147.0241	5000	10000
2089703	147.3595	11.80165	114.2423	193.9736	79.73138	0.327198	147.6867	147.0323	5000	10000
2178625	147.3642	11.57235	109.0153	202.2769	93.26154	0.320841	147.685	147.0433	5000	10000
2165011	147.3949	11.6111	111.9185	214.3246	102.406	0.321915	147.7168	147.073	5000	10000
2037363	147.3996	11.58961	114.701	200.7663	86.06528	0.32132	147.7209	147.0783	5000	10000
2163475	147.4079	11.54231	115.0559	203.1759	88.11999	0.320008	147.7279	147.0879	5000	10000
2045639	147.4202	11.78553	110.1529	205.8718	95.71891	0.326751	147.7469	147.0934	5000	10000
2126835	147.4204	11.64507	116.0197	213.8922	97.87244	0.322857	147.7433	147.0975	5000	10000
2139495	147.4224	11.74234	115.1794	199.5605	84.38103	0.325554	147.7479	147.0968	5000	10000
2112199	147.4234	11.53013	118.1207	208.9004	90.77974	0.31967	147.743	147.1037	5000	10000
2081119	147.4255	11.67194	113.3359	204.4063	91.07031	0.323602	147.7491	147.1019	5000	10000
2102568	147.4266	11.60425	110.9229	205.2666	94.34367	0.321725	147.7483	147.1048	5000	10000
2149937	147.4303	11.53551	111.1924	212.4788	101.2864	0.31982	147.7501	147.1105	5000	10000
2104554	147.4315	11.77844	116.3755	205.8301	89.45458	0.326555	147.758	147.1049	5000	10000
2019290	147.434	11.68408	113.0394	194.2851	81.24565	0.323939	147.758	147.1101	5000	10000
2184225	147.4347	11.84522	112.4988	199.3671	86.8683	0.328406	147.7631	147.1063	5000	10000
2042486	147.4358	11.62266	108.6649	204.8288	96.16396	0.322236	147.758	147.1136	5000	10000
2105721	147.4361	11.62298	110.4442	205.7964	95.35222	0.322245	147.7584	147.1139	5000	10000
2103182	147.4407	11.69794	113.7317	204.4957	90.76395	0.324323	147.765	147.1164	5000	10000
2177037	147.4419	11.54768	109.6516	196.7576	87.10605	0.320157	147.762	147.1217	5000	10000
2080121	147.4498	11.32216	110.3133	196.6638	86.3505	0.313904	147.7637	147.1359	5000	10000

$$\text{seed} = \text{frac}(10000 * \text{abs}(\sin(\text{seed} * \pi + 1.618)))$$

- Suggested by ChatGPT
- Factor is about 150
- Replacing 10000 with 13579 yields a Factor of 151 to 152.

Single-Seed Nested Exponential PRNGs

- Uses single seed r .
- Uses intermediate variables x_1 and x_2 as expressions of r , such that x_1 and x_2 are always at least equal to 0.
- Based on variants of $\textit{frac}(\exp(\exp(\exp(r))))$

$$\text{frac}(\exp(x1+\exp(x1)+\exp(x2)))$$

- Ranks #1
- $x1=1-r+r^2-r^3$ (value in range (0,1))
- $x2=1+r-r^2$ (values $\Rightarrow 1$ and < 2)
- r is the current random number
- Factor = 146.100052, sdev = 12.416211

Code for $\text{frac}(\exp(x1+\exp(x1))+\exp(x2)))$

- r = initial or current seed
- $x1 = 1 - r + r^2 - r^3$
- $x2 = 1 + r - r^2$
- $r = \text{frac}(\exp(x1+\exp(x1))+\exp(x2)))$

$$\text{frac}(\exp(1/x1 + \exp(x1) + \exp(x2)))$$

- Ranks # 2
- $x1 = 1 + r$ (values in range (1,2))
- $x2 = 1 + r - r^2 - r^3$ (values in range (0,1))
- r is the current random number
- Factor = 146.606776 , sdev = 14.777862

$$\text{frac}(\exp(1/x2 + \exp(x1)))$$

- Ranks # 3
- $x1 = 1 + r$ (values in range (1,2))
- $x2 = 1 + r^2$ (values in range (1,2))
- r is the current random number
- Factor = 146.8799, sdev = 12.30304

$$\text{frac}(\exp(\textcolor{red}{x2} + \exp(x1) + \exp(x2)))$$

- Ranks # 4
- $x1 = 1 + r^2$ (values in range (1,2))
- $x2 = 1 + r - r^2$ (values $\Rightarrow 1$ and < 2)
- r is the current random number
- Factor = 146.918401, sdev = 12.163099

$$\text{frac}(\exp(\text{sqrt}(x2))+\exp(x1)))$$

- Ranks # 5
- $x1=1-r-r^2+r^3$ (values in range (0,1))
- $x2=1+r-r^2-r^3$ (values in range (0,1))
- r is the current random number
- Factor = 146.930115, sdev = 16.030966

Program	mean	sdev	S3	S1	s2
doPC7	146.9471	13.3651	$\exp(x_1+x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r+r^2-r^3$	$x_2=1+r-r^2$
doPC7	146.9479	12.40661	$\exp(1/\sqrt{x_1}+x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r+r^2-r^3$	$x_2=1+r-r^2-r^3$
doPC1	146.9705	11.83723	$\exp(1/\sqrt{x_1}+1/x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r$	$x_2=1-r+r^2$
doPC3	147.0156	11.4665	$\exp(1/\sqrt{x_1}+\exp(x_2))$	$x_1=1+r-r^2$	$x_2=1+r$
doPC7	147.026	11.7997	$\exp(x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r+r^2-r^3$	$x_2=1+r-r^2$
doPC2	147.0658	12.59195	$\exp(1/x_1+\exp(x_2))$	$x_1=1+r^2$	$x_2=1+r$
doPC3	147.0758	11.95597	$\exp(\sqrt{x_1}+x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r-r^2$	$x_2=1+r$
doPC3	147.1226	11.51817	$\exp(x_1+x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r-r^2$	$x_2=1+r^2$
doPC2	147.1519	11.54015	$\exp(1/\sqrt{x_2}+\exp(x_1)+\exp(x_2))$	$x_1=1+r^2$	$x_2=1+r-r^2$
doPC10	147.2444	18.34358	$\exp(\sqrt{x_1}+x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r-r^2-r^3$	$x_2=1+r^2$
doPC6	147.2742	11.63198	$\exp(1/\sqrt{x_1}+\exp(x_1)+\exp(x_2))$	$x_1=1+r-r^2+r^3$	$x_2=1+r^2$
doPC10	147.2859	11.7676	$\exp(x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r-r^2-r^3$	$x_2=1+r^2$
doPC6	147.294	12.08173	$\exp(x_1+1/x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r-r^2+r^3$	$x_2=1+r^2$

Program	mean	sdev	S3	S1	s2
doPC10	147.3206	17.41561	$\exp(\sqrt{x_1} + \exp(x_2))$	$x_1 = 1 + r - r^2 - r^3$	$x_2 = 1 - r - r^2 + r^3$
doPC5	147.3515	11.36086	$\exp(1/x_1 + x_2 + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r + r^2 + r^3$	$x_2 = 1 + r - r^2$
doPC6	147.3789	12.8636	$\exp(1/\sqrt{x_1} + \exp(x_1) + \exp(x_2))$	$x_1 = 1 + r - r^2 + r^3$	$x_2 = 1 + r$
doPC8	147.4869	11.79038	$\exp(x_1 + \sqrt{x_2} + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r + r^2 - r^3$	$x_2 = 1 + r - r^2 - r^3$
doPC8	147.5104	11.74795	$\exp(\sqrt{x_1} + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r + r^2 - r^3$	$x_2 = 1 - r + r^2 - r^3$
doPC4	147.5513	11.56876	$\exp(x_2 + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r + r^2$	$x_2 = 1 + r$
doPC9	147.5672	11.82622	$\exp(x_1 + \sqrt{x_2} + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r - r^2 + r^3$	$x_2 = 1 + r - r^2$
doPC5	147.6139	24.21378	$\exp(1/\sqrt{x_1} + x_2 + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r + r^2 + r^3$	$x_2 = 1 + r$
doPC5	147.6163	13.22974	$\exp(1/\sqrt{x_1} + \exp(x_2))$	$x_1 = 1 - r + r^2 + r^3$	$x_2 = 1 + r$
doPC9	147.663	11.89951	$\exp(x_1 + 1/\sqrt{x_2} + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r - r^2 + r^3$	$x_2 = 1 + r - r^2$
doPC4	147.7918	13.0267	$\exp(x_1 + \sqrt{x_2} + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r + r^2$	$x_2 = 1 - r + r^2 - r^3$
doPC4	147.8255	12.45127	$\exp(1/x_1 + x_2 + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r + r^2$	$x_2 = 1 + r$

Multiple-Seed Exponential PRNG

- Uses two seeds r_1 and r_2 .
- Uses intermediate variables x_1 and x_2 as expressions of r_1 and r_2 , such that x_1 and x_2 are always at least equal to 0.

$$\text{frac}(\exp(x1+x2+\exp(x1)+\exp(x2)))$$

- Ranks # 1
- $x1=1+r1+r1^2-r1^3$. (values in range (1,2))
- $x2=1-r2+r2^2-r2^3$. (values in range (0,1))
- r2 and r1 is the current and previous random numbers
- Factor = 147.250251, sdev = 11.642747

Code for $\text{frac}(\exp(x_1+x_2+\exp(x_1)+\exp(x_2)))$

- r_1 = previous or initial seed
- r_2 = current or initial seed
- $r_{20} = r_2$
- $x_1 = 1 + r_1 + r_1^2 - r_1^3$.
- $x_2 = 1 - r_2 + r_2^2 - r_2^3$
- $r_1 = r_{20}$
- $r_2 = \exp(x_1 + x_2 + \exp(x_1) + \exp(x_2))$

$$\text{frac}(\exp(1/x_1 + x_2 + \exp(x_1) + \exp(x_2)))$$

- Ranks # 2
- $x_1 = 1 - r_1 + r_1^2$. (values $\Rightarrow 1$ and < 2)
- $x_2 = 1 + r_2^2$. (values in range $(1, 2)$)
- r_2 and r_1 is the current and previous random numbers
- Factor = 147.282169, sdev = 11.863594

$$\text{frac}(\exp(x1+x2+\exp(x1)+\exp(x2)))$$

- Ranks # 3
- $x1=1+r1$. (values in range (1,2))
- $x2=1+r2^2$. (values in range (1,2))
- r2 and r1 is the current and previous random numbers
- Factor = 147.321515, sdev = 11.627948

$$\text{frac}(\exp(x1+x2+\exp(x1)+\exp(x2)))$$

- Ranks # 4
- $x1=1-r1-r1^2+r1^3$. (values in range (0,1))
- $x2=1+r2-r2^2$. (values $\Rightarrow$ 1 and < 2)
- $r2$ and $r1$ is the current and previous random numbers
- Factor = 147.34052, sdev = 11.815902

$$\text{frac}(\exp(\sqrt{x1})+\exp(x2)))$$

- Ranks # 5
- $x1=1+r1+r2-r2^2$. (values in range (1,2))
- $x2=1+r2+r1-r1^2$. (values in range (1,2))
- r2 and r1 is the current and previous random numbers
- Factor = 147.35175, sdev = 11.667883

Program	mean	sdev	s1	s2	s3
doPC07	147.35204	11.699046	$\exp(1/\sqrt{x_1}+1/x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r_1+r_1^2-r_1^3$;	$x_2=1+r_2$;
doPC04	147.35932	11.665847	$\exp(\sqrt{x_1}+x_2+\exp(x_1)+\exp(x_2))$	$x_1=1-r_1+r_1^2$;	$x_2=1+r_2-r_2^2-r_2^3$;
doPC11	147.36445	11.536546	$\exp(1/x_1+x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r_2-r_1+r_1^2$	$x_2=1+r_1-r_2+r_2^2$
doPC06	147.37643	11.660359	$\exp(x_2+\exp(x_1))$	$x_1=1+r_1-r_1^2+r_1^3$;	$x_2=1+r_2-r_2^2$;
doPC03	147.38013	11.834986	$\exp(x_1+\exp(x_2))$	$x_1=1+r_1-r_1^2$;	$x_2=1+r_2$;
doPC07	147.40169	11.78991	$\exp(x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r_1+r_1^2-r_1^3$;	$x_2=1+r_2-r_2^2-r_2^3$;
doPC06	147.4021	11.789396	$\exp(\sqrt{x_2}+\exp(x_1)+\exp(x_2))$	$x_1=1+r_1-r_1^2+r_1^3$;	$x_2=1-r_2+r_2^2-r_2^3$;
doPC03	147.40222	11.539933	$\exp(x_1+x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r_1-r_1^2$;	$x_2=1-r_2-r_2^2+r_2^3$;
doPC02	147.40327	11.834294	$\exp(1/x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r_1^2$;	$x_2=1+r_2-r_2^2$;
doPC03	147.41522	11.817072	$\exp(x_1+x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r_1-r_1^2$;	$x_2=1+r_2^2$;
doPC10	147.42201	11.694754	$\exp(\sqrt{x_1}+x_2+\exp(x_1)+\exp(x_2))$	$x_1=1+r_1-r_1^2-r_1^3$;	$x_2=1+r_2$;
doPC08	147.42577	11.716497	$\exp(x_1+\exp(x_1)+\exp(x_2))$	$x_1=1-r_1+r_1^2-r_1^3$;	$x_2=1+r_2-r_2^2-r_2^3$;
doPC04	147.43035	11.564335	$\exp(\sqrt{x_2}+\exp(x_1)+\exp(x_2))$	$x_1=1-r_1+r_1^2$;	$x_2=1+r_2^2$;
doPC05	147.4433	11.796697	$\exp(x_1+\exp(x_1)+\exp(x_2))$	$x_1=1-r_1+r_1^2+r_1^3$;	$x_2=1+r_2$;

Program	mean	sdev	s1	s2	s3
doPC02	147.44447	11.726875	$\exp(x_2 + \exp(x_1) + \exp(x_2))$	$x_1 = 1 + r_1^2;$	$x_2 = 1 - r_2 + r_2^2;$
doPC02	147.44834	11.808034	$\exp(x_2 + \exp(x_1))$	$x_1 = 1 + r_1^2;$	$x_2 = 1 - r_2 + r_2^2 - r_2^3;$
doPC10	147.45044	11.742755	$\exp(\sqrt{x_1} + x_2 + \exp(x_1) + \exp(x_2))$	$x_1 = 1 + r_1 - r_1^2 - r_1^3;$	$x_2 = 1 + r_2 - r_2^2;$
doPC10	147.45129	11.887212	$\exp(x_1 + \exp(x_1) + \exp(x_2))$	$x_1 = 1 + r_1 - r_1^2 - r_1^3;$	$x_2 = 1 - r_2 + r_2^2 - r_2^3;$
doPC09	147.45178	11.705228	$\exp(1/x_2 + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r_1 - r_1^2 + r_1^3;$	$x_2 = 1 + r_2;$
doPC05	147.45272	11.627955	$\exp(1/x_2 + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r_1 + r_1^2 + r_1^3;$	$x_2 = 1 + r_2;$
doPC06	147.45955	11.943712	$\exp(1/x_1 + \exp(x_1) + \exp(x_2))$	$x_1 = 1 + r_1 - r_1^2 + r_1^3;$	$x_2 = 1 - r_2 + r_2^2 - r_2^3;$
doPC01	147.4624	11.756058	$\exp(1/x_1 + x_2 + \exp(x_1) + \exp(x_2))$	$x_1 = 1 + r_1;$	$x_2 = 1 - r_2 + r_2^2;$
doPC05	147.46417	11.947129	$\exp(\sqrt{x_1} + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r_1 + r_1^2 + r_1^3;$	$x_2 = 1 + r_2;$
doPC01	147.46468	11.873286	$\exp(x_1 + \exp(x_2))$	$x_1 = 1 + r_1;$	$x_2 = 1 - r_2 + r_2^2;$
doPC09	147.50744	11.558537	$\exp(x_1 + x_2 + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r_1 - r_1^2 + r_1^3;$	$x_2 = 1 + r_2 - r_2^2 - r_2^3;$
doPC11	147.52809	11.712875	$\exp(x_2 + \exp(x_1))$	$x_1 = 1 + r_1 - r_1^2 - r_1^3;$	$x_2 = 1 - r_2 + r_2^2 - r_2^3;$
doPC08	147.52922	11.568263	$\exp(\sqrt{x_1} + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r_1 + r_1^2 - r_1^3;$	$x_2 = 1 + r_2 - r_2^2 - r_2^3;$
doPC08	147.53917	11.597979	$\exp(x_1 + x_2 + \exp(x_1) + \exp(x_2))$	$x_1 = 1 - r_1 + r_1^2 - r_1^3;$	$x_2 = 1 - r_2 + r_2^2 - r_2^3;$

Conclusion

- All algorithms are good.
- The single-seed nested exponentials have a slight lead over the other two sets of algorithms.

Thank You!